THE NCTM PROCESS STANDARDS

Problem Solving

Instructional programs from prekindergarten through grade 12 should enable all students to--

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

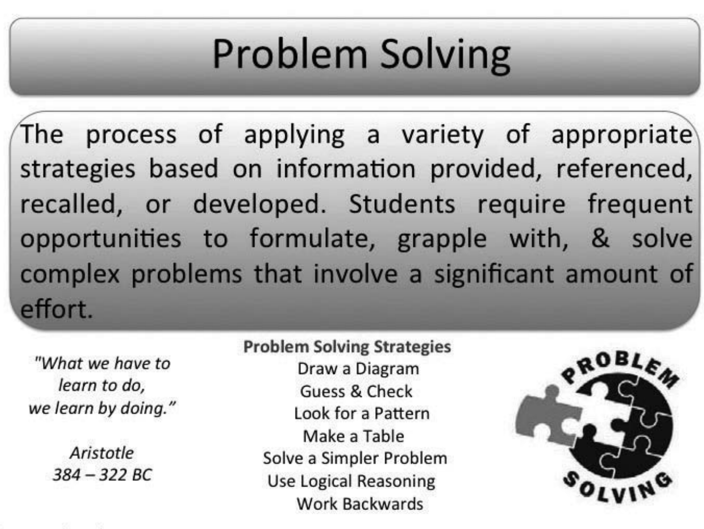
Problem solving is an integral part of all mathematics learning. In everyday life and in the workplace, being able to solve problems can lead to great advantages. However, solving problems is not only a goal of learning mathematics but also a major means of doing so. Problem solving should not be an isolated part of the curriculum but should involve all Content Standards.

Problem solving means engaging in a task for which the solution is not known in advance. Good problem solvers have a "mathematical disposition"--they analyze situations carefully in mathematical terms and naturally come to pose problems based on situations they see. For example, a young child might wonder, How long would it take to count to a million?

Good problems give students the chance to solidify and extend their knowledge and to stimulate new learning. Most mathematical concepts can be introduced through problems based on familiar experiences coming from students' lives or from mathematical contexts. For example, middle-grades students might investigate which of several recipes for punch giving various amounts of water and juice is "fruitier." As students try different ideas, the teacher can help them to converge on using proportions, thus providing a meaningful introduction to a difficult concept.

Students need to develop a range of strategies for solving problems, such as using diagrams, looking for patterns, or trying special values or cases. These strategies need instructional attention if students are to learn them. However, exposure to problem-solving strategies should be embedded across the curriculum. Students also need to learn to monitor and adjust the strategies they are using as they solve a problem.

Teachers play an important role in developing students' problem-solving dispositions. They must choose problems that engage students. They need to create an environment that encourages students to explore, take risks, share failures and successes, and question one another. In such supportive environments, students develop the confidence they need to explore problems and the ability to make adjustments in their problem-solving strategies.



Reasoning and Proof

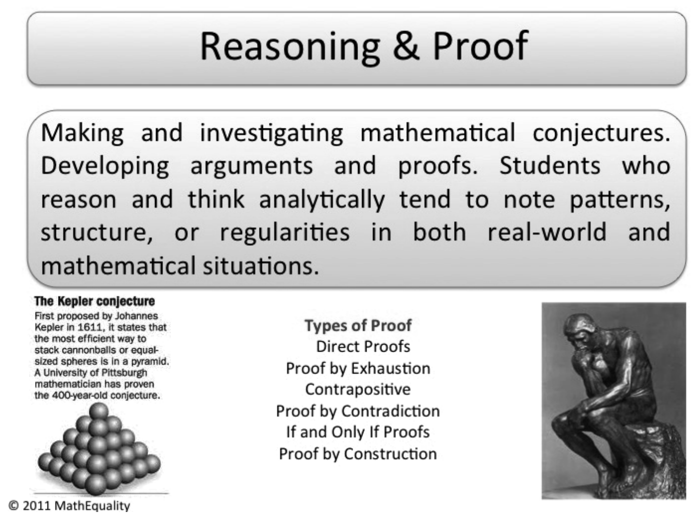
Instructional programs from prekindergarten through grade 12 should enable all students to--

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Systematic reasoning is a defining feature of mathematics. Exploring, justifying, and using mathematical conjectures are common to all content areas and, with different levels of rigor, all grade levels. Through the use of reasoning, students learn that mathematics makes sense. Reasoning and proof must be a consistent part of student's mathematical experiences in prekindergarten through grade 12.

Reasoning mathematically is a habit of mind, and like all habits, it must be developed through consistent use in many contexts and from the earliest grades. At all levels, students reason inductively from patterns and specific cases. For example, even a first grader can use an informal proof by contradiction to argue that the number 0 is even: "If 0 were odd, then 0 and 1 would be two odd numbers in a row. But even and odd numbers alternate. So 0 must be even."

Increasingly over the grades, students should learn to make effective deductive arguments as well, using the mathematical truths they are establishing in class. By the end of secondary school, students should be able to understand and produce some mathematical proofs--logically rigorous deductions of conclusions from hypotheses--and should appreciate the value of such arguments.



Communication

Instructional programs from prekindergarten through grade 12 should enable all students to--

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

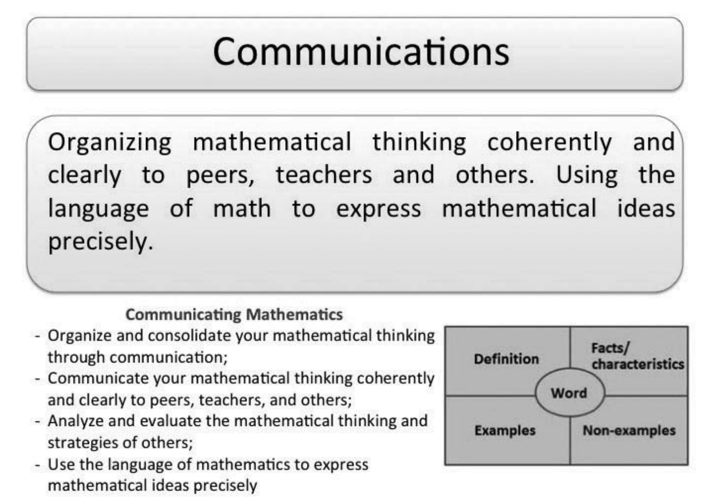
As students are asked to communicate about the mathematics they are studying--to justify their reasoning to a classmate or to formulate a question about something that is puzzling--they gain insights into their thinking. In order to communicate their thinking to others, students naturally reflect on their learning and organize and consolidate their thinking about mathematics.

Students should be encouraged to increase their ability to express themselves clearly and coherently. As they become older, their styles of argument and dialogue should more closely adhere to established conventions, and students should become more aware of, and responsive to, their audience. The ability to write about mathematics should be particularly nurtured across the grades.

By working on problems with classmates, students also have opportunities to see the perspectives and methods of others. They can learn to understand and evaluate the thinking of others and to build on those ideas. For example, students who try to solve the following problem algebraically may have difficulty setting up the equations:

There are some rabbits and some hutches. If one rabbit is put in each hutch, one rabbit will be left without a hutch. If two rabbits are put in each hutch, one hutch will remain empty. How many rabbits and how many hutches are there?

They may benefit from the insights of students who solve the problem using a visual representation. Students need to learn to weigh the strengths and limitations of different approaches, thus becoming critical thinkers about mathematics.



Connections

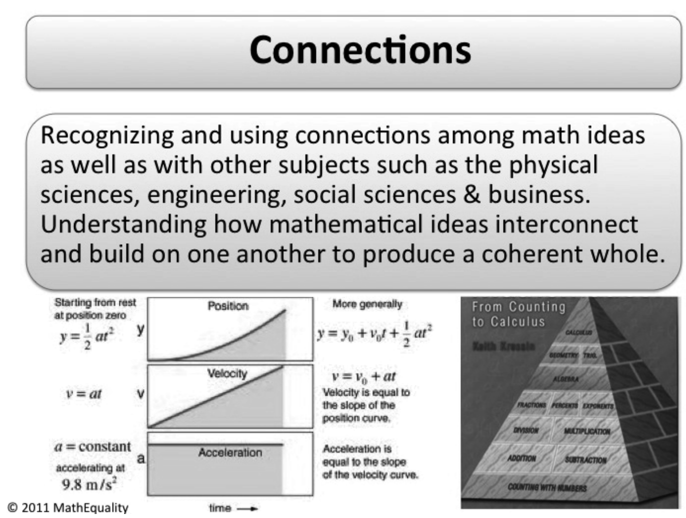
Instructional programs from prekindergarten through grade 12 should enable all students to--

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Mathematics is an integrated field of study, even though it is often partitioned into separate topics. Students from prekindergarten through grade 12 should see and experience the rich interplay among mathematical topics, between mathematics and other subjects, and between mathematics and their own interests. Viewing mathematics as a whole also helps students learn that mathematics is not a set of isolated skills and arbitrary rules.

An emphasis on mathematical connections helps students recognize how ideas in different areas are related. Students should come both to expect and to exploit connections, using insights gained in one context to verify conjectures in another. For example, elementary school students link their knowledge of the subtraction of whole numbers to the subtraction of decimals or fractions. Middle school students might collect and graph data for the circumference (C) and diameter (d) of various circles. They could extend their previous knowledge in algebra and data analysis to recognize that the values nearly form a straight line, so C/d is between 3.1 and 3.2 (a rough estimation of π).

The opportunity to experience mathematics in context is important. Students should connect mathematical concepts to their daily lives, as well as to situations from science, the social sciences, medicine, and commerce. For example, high school students worked with a drug store chain to determine where it should locate a new pharmacy in their neighborhood on the basis of analyses of demographic and economic data. Students should recognize the value of mathematics in examining personal and societal issues.



Representation

Instructional programs from prekindergarten through grade 12 should enable all students to--

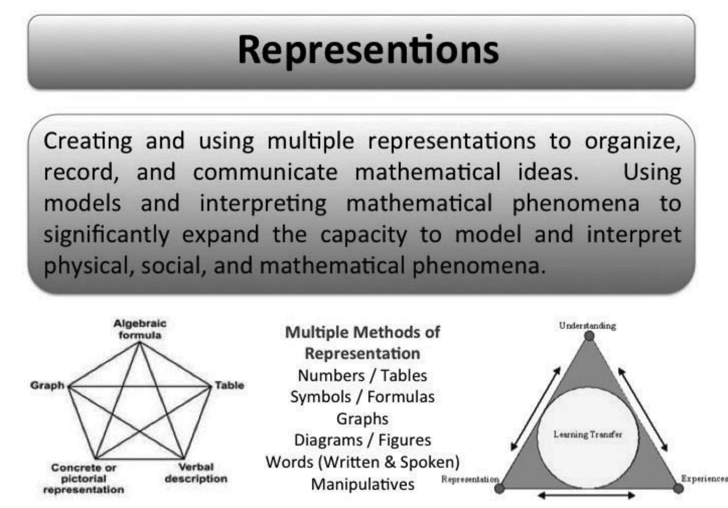
- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

Representations are necessary to students' understanding of mathematical concepts and relationships. Representations allow students to communicate mathematical approaches, arguments, and understanding to themselves and to others. They allow students to recognize connections among related concepts and apply mathematics to realistic problems.

To become deeply knowledgeable about fractions, for example, students need a variety of representations that support their understanding. They need to understand various interpretations of fractions, such as ratio, indicated division, or fraction of a number. They need to understand other common representations for fractions, such as points on a number line.

Some forms of representation--such as diagrams, graphical displays, and symbolic expressions--have long been part of school mathematics. Unfortunately, these representations and others have often been taught and learned as if they were ends in themselves. This approach limits the power and utility of representations as tools for learning and doing mathematics.

It is important to encourage students to represent their mathematical ideas in ways that make sense to them, even if those representations are not conventional. At the same time, students should learn conventional forms of representation in ways that facilitate their learning of mathematics and their communication with others about mathematical ideas. The integration of technology into mathematics instruction further increases the need for students to be comfortable with new mathematical representations.



Process Education™ — Past, Present, and Future

Kathleen Burke[1], Betty Lawrence[2], Mohamed El-Sayed[3], and Dan Apple[4]

Abstract

Process Education (PE), a term that came into being twenty-five years ago, is built upon a foundation of educational philosophies and approaches centered in active and facilitated learning. As background, we define Process Education, as well as identify some of its underlying concepts and related efforts. We describe the evolution of Process Education and provide examples of its impact within higher education. Finally, we explore potential future directions of Process Education.

Introduction

> Process Education™ can be defined as a **performance-based** philosophy of education which integrates many different educational **theories**, **processes**, and **tools** in emphasizing the continuous development of **learning skills** through the use of **assessment** principles in order to produce **learner self-development**.
>
> (http://www.pcrest.com/PC/PE/index.html)

Process Education™ (PE) principles are founded on two basic beliefs. The first is that every learner can learn to learn better, regardless of his or her current level of achievement; one's potential is not limited by current ability. The second principle is that educators have a responsibility to "raise the bar" in their profession because learning is enhanced and achieved for all learners when educators help build learning skills, create and improve quality learning environments, design solid coherent curricula, and serve as effective facilitators of learning.

PE requires that learning and facilitation of learning take place within an assessment culture, rather than a culture of evaluation. In the traditional educational model, the focus is upon evaluation—an educator judges a student's efforts and performance against an objective criteria with standards. While this evaluation can provide a useful snapshot of performance, it does not encourage the *improvement* of that performance. Through the careful use of assessment, however, students can continually improve the quality of their performance. This is critical, as the goal of PE is to help individuals develop into self-growers. Self-growers are learners who seek to improve their own learning performance; can create their own challenges; serve as leaders and mentors to others; take control of their own destiny, and self-assesses and self-mentors to facilitate their own growth.

As this paper aims to present a comprehensive introduction to Process Education, we will briefly survey its philosophical underpinnings, examine the evolution and impact of PE over the last 25 years, and finally consider possible avenues for PE growth and application in the future.

Philosophical Underpinnings and Efforts Related to Process Education

The word *education* usually refers to the process of gaining or cultivating knowledge, skills, beliefs, attitudes, values, and character traits. Traditional educational philosophies were profoundly influenced by the thinking and teachings of individuals such as Plato, Aristotle, Augustine, and John Locke. Beginning in roughly the later half of the twentieth century, educational philosophies were increasingly developed and articulated in the contexts of different disciplines (e.g. educational history, sociology, psychology), rather than the context of any particular philosophical school (Frankena, 1971). As a result, educational philosophy has evolved from a historically narrow field to a kind of broad category, containing a multiplicity of different perspectives.

Process Education is based upon a foundation of several different educational philosophies and approaches, most of which fall into the general category of constructivism. Constructivism is built upon the cognitive theory of development as pioneered by Jean Piaget. One of the core assumptions of constructivism is that learning is an active, contextualized process of constructing rather than acquiring knowledge. This knowledge is constructed on the basis of personal experiences and the hypotheses that a learner makes about the environment. Piaget is also credited with identifying stages of (largely cognitive) learner development. Subsequent theorists built on or provided alternatives to his ideas. Lev Vygotsky's social developmental theory, for example, focused more heavily on the influence of social interaction in the process of cognitive development. Jerome Bruner also looked to

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environmental and experiential factors. His book, *The Process of Education*, built on constructivism, especially the structure of learning and learning readiness, leading to his recommendation of approaches such as a spiral curriculum and discovery learning.

Discovery, Experience, and the Role of the Educator

Discovery learning, also known as inquiry-based learning, builds on the ideas of John Dewey, Piaget and other constructivists. Dewey wrote (1938), "There is an intimate and necessary relation between the processes of actual experience and education." Through discovery learning, the learner is placed in situations whereby he or she calls on prior knowledge and past experience to discover new information or skills. Discovery learning situations can range from the unstructured and open to those carefully structured by a facilitator in order to lead a learner to a planned destination.

> Emphasis on discovery in learning has precisely the effect on the learner of leading him to be a constructionist, to organize what he is encountering in a manner not only designed to discover regularity and relatedness, but also to avoid the kind of information drift that fails to keep account of the uses to which information might have to be put. (Bruner, 1962)

Educational theorists like Alan Tough and Malcolm Knowles have applied these concepts to adults, using the term *self-directed learning*. Self-directed learning has become increasingly important as our rapidly changing world necessitates life-long learning, extending well beyond any formal classroom. Knowles was an especially strong advocate for the self-directed learner, arguing that proactive learners enter into learning more purposefully and with greater motivation, leading to increased retention (Knowles, 1975).

Educational theorist David Kolb spoke of the benefits of learning from experience. He proposed a learning cycle, which starts with a concrete experience, proceeds to observation and reflection on that experience, moves on to forming abstract concepts based on the reflection, and ends with testing these concepts in new situations (Kolb, 1975). Experiential education emerged from his ideas, which, according to the Association for Experiential Education, is defined as, "a philosophy and methodology in which educators purposefully engage with learners in direct experience and focused reflection in order to increase knowledge, develop skills and clarify values" (www.aee.org). Table 1 offers the Principles of Experiential Education according to the Association of Experiential Education. It is noteworthy that proponents of service learning embrace many of these principles as well (www.servicelearning.org).

Table 1 Principles of Experiential Education

- Experiential learning occurs when carefully chosen experiences are supported by reflection, critical analysis and synthesis.
- Experiences are structured to require the learner to take initiative, make decisions and be accountable for results.
- Throughout the experiential learning process, the learner is actively engaged in posing questions, investigating, experimenting, being curious, solving problems, assuming responsibility, being creative, and constructing meaning.
- Learners are engaged intellectually, emotionally, socially, soulfully and/or physically. This involvement produces a perception that the learning task is authentic.
- The results of the learning are personal and form the basis for future experience and learning.
- Relationships are developed and nurtured: learner to self, learner to others and learner to the world at large.
- The educator and learner may experience success, failure, adventure, risk-taking and uncertainty, because the outcomes of experience cannot totally be predicted.
- Opportunities are nurtured for learners and educators to explore and examine their own values.
- The educator's primary roles include setting suitable experiences, posing problems, setting boundaries, supporting learners, insuring physical and emotional safety, and facilitating the learning process.
- The educator recognizes and encourages spontaneous opportunities for learning.
- Educators strive to be aware of their biases, judgments and pre-conceptions, and how these influence the learner.
- The design of the learning experience includes the possibility to learn from natural consequences, mistakes and successes.

Source: www.aee.org

Process Education shares many components with both experiential education and problem-based learning, or PBL, another active and learner-centered approach to education. (www.pbl.org). PBL was introduced at McMaster University and was documented extensively by Barrows and Tamblyn, who applied it to medical education, where faculty were frustrated with the effectiveness of traditional teaching methods. Barrows and Tamblyn found that medical school graduates were often not able to apply knowledge they had learned to the experiential challenges they faced when working as interns in a hospital environment.

Through PBL, students are presented with an ill-defined problem and they work cooperatively to solve the problem, accessing resources as needed. An important aspect of PBL is that it is student-centered, with the students, rather than the instructor, managing the problem-solving process. The faculty member in PBL serves as a facilitator of learning.

Central to each of the methods previously described is the role of the faculty member as a facilitator of the learning process. There are many different strategies for facilitative learning; the main goal of each is to move the teacher away from the center and locus of control.

Cooperative Learning, Mentoring, and Learning Communities

Much has been written about the use of cooperative learning in education. As Wong and Wong stated in 1998, "Cooperative learning is not so much learning to cooperate as it is cooperating to learn." As they and others have indicated, cooperative learning extends far deeper than just placing students in groups. Two central elements of cooperative learning are positive interdependence and both group and individual accountability.

The concept of mentoring is increasingly accepted as a valid and promising model for increasing student learning. Traditionally, the mentor has been seen as the "sage," (King, 1993) but more recent formulations have positioned the mentor as more equal to the learner and as one who also learns from the interaction. The mentor does, however, engage with the learner in what is sometimes termed "authentic assessment" or "performance-based assessment." These strategies draw on the approaches of PBL and experiential learning. Assuming a learner is placed at the center of the learning experience, different strategies are needed to assess his or her performance. The facilitator, or mentor, works with the student to identify his or her level of performance. Rubrics are often used to assist in the identification of these levels.

The mentor, or facilitator of learning, may also utilize instructional scaffolding to assist the learner, an concept articulated by Bruner. In scaffolding, the task is adjusted according to the current level of the student. Bruner spoke of a spiral curriculum, meaning that the learner is guided from level to level by carefully building on previous learning experiences. Scaffolding is also an aspect of the approach of differentiated instruction, where the teacher adjusts the learning situation to the learner, rather than imposing a one-size-fits-all curriculum on students.

Related to the approaches of both facilitated and cooperative learning is the valuing of the learning community. Many have looked to the writings of Paulo Freire who articulated the importance of dialogue, where, rather than one person acting upon another, individuals work with one another in a community.

As an educational philosophy *Process Education* is a synthesis of realist and idealist world views, with a primary focus on performance. It integrates many of the tenets of constructivism with personal development, performance measures, and assessment in order to produce learner growth, promote critical thinking, and nurture continuous improvement.

The Evolution of Process Education

In 1985, Pacific Crest began introducing its software, *PC:Solve*, to institutions of higher learning across the country. They conducted small workshops demonstrating how students were able to independently learn to use the software by processing the information presented within the software's help system. The students were tasked with critically reading this information in order to gain an understanding of the given examples. To succeed, the students needed to take risks and try things out. Through the use of analysis and synthesis they would apply the appropriate tools to the problems presented. Finally, Pacific Crest demonstrated (to the faculty observing) how students were able to generalize and transfer skills to apply what they were learning to new situations. Within the following years, Pacific Crest added reflection and self-assessment to this process so that the metacognition of what was happening was apparent to the students themselves. These informal self-assessments allowed the students to reflect on their learning which helped to improve their future learning and performance.

Between 1989 and 1990 Pacific Crest conducted an empirical study of 22 colleges from across the country. These institutions included an engineering college, a business college, a women's college, a highly selective research university, as well as several liberal arts, state and technical colleges. At each institution a random sample of seniors, juniors, sophomores, and freshmen were selected by faculty and a competition was set up matching seniors against each of the other three class levels. The students were asked to perform an array of challenging tasks that required them to think about information critically, process it, and transfer it to new situations. The faculty observed their students perform these tasks for a 90 to 120 minute period. By the end of that period, many faculty were frustrated and often very disappointed with their seniors, because the seniors' performance was not significantly better than the performance of freshmen. These outcomes, replicated again and again, convinced Pacific Crest that current practices within higher education were not helping students develop life-long learning skills, as learner performance was not found to be significantly increased over four years of college.

This action research helped Pacific Crest develop two key resources. The first was the Learning Process Methodology (LPM). The LPM is a relatively generic model for learning; it is content-independent and can be applied to nearly any learning situation. The potential of the LPM is that it can be used to teach students *how to learn*, as the LPM makes the learning process itself concrete and accessible to a learner. The second key resource was the Classification of Learning Skills (CLS). This organized list identified transferable skills that could be used in any learning context. The initial list included skills such as listening, persisting, transferring, and articulating an idea. The potential of the CLS is that in strengthening learning skills, learners not only learn content more efficiently and successfully, but also become better at the task of learning, itself.

In 1991, Pacific Crest held its first "Problem Solving across the Curriculum" conference. The conference was attended by more than 100 faculty from various disciplines. The faculty set out to define a set of practices and approaches that would *empower* students to succeed. Many of the conversations regarding these practices lasted until the early hours of the morning. These discussions marked the beginning of an explicit philosophy of Process Education and inspired an annual meeting for this conference.

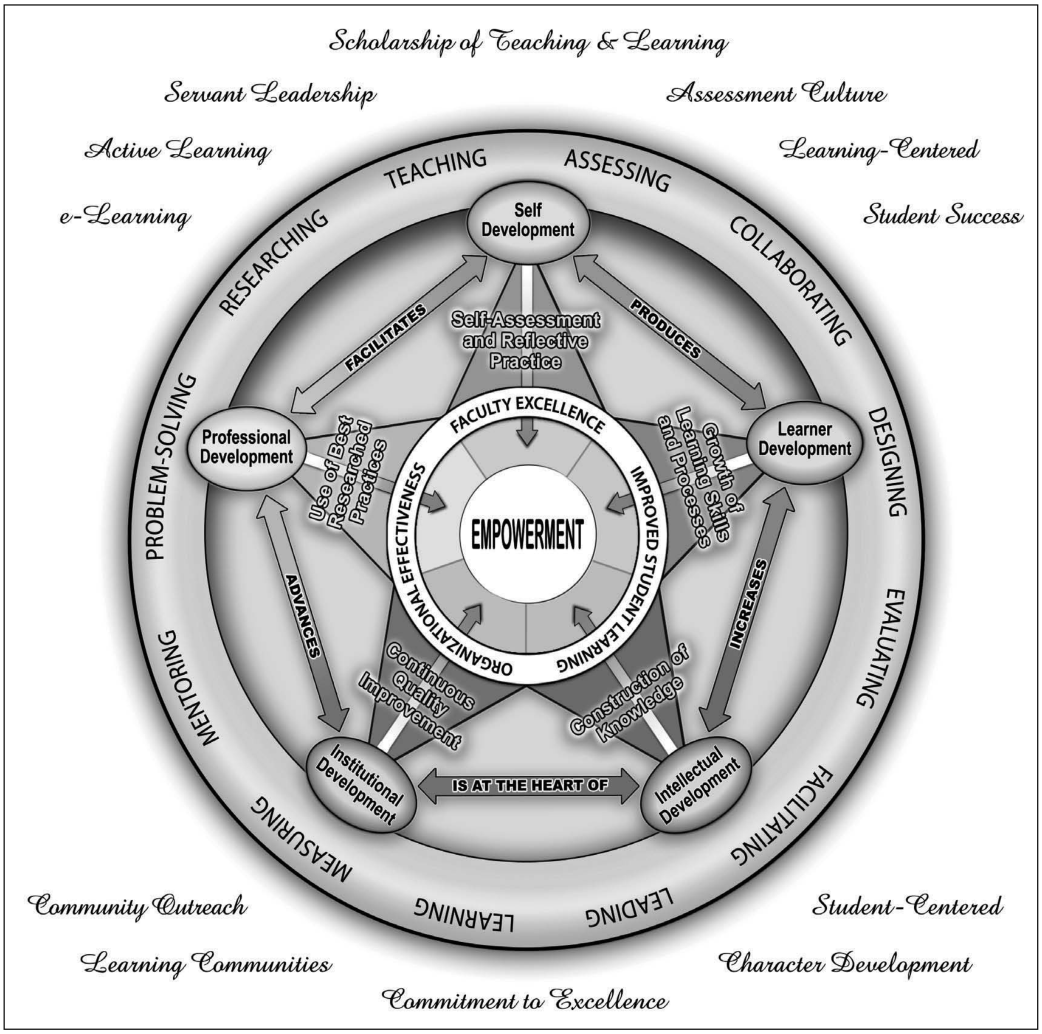
These practices and approaches were first implemented in 1994 at the first Learning-to-Learn Camp. This camp was geared toward a population of college students identified as "at-risk." The goal of the camp was to prove that all students could learn to meet the college's performance expectations and graduate with success. Over the course of a single week, all parties involved in the first Learning-to-Learn Camp began to understand how potentially powerful Process Education was. They observed as the application of PE principles began to literally transform individual lives, despite the fact that the supporting practices that currently existed for these camps were still in their infancy and a bit rough around the edges.

In 1994, Betty Lawrence and Dan Apple presented the paper "Education as a Process" at the International

Table 2 The Ten Principles of Process Education

1. Every learner can learn to learn better, regardless of current level of achievement; one's potential is not limited by current ability.
2. Although everyone requires help with learning at times, the goal is to become a capable, self-sufficient, lifelong learner.
3. An empowered learner is one who uses learning processes and self-assessment to improve future performance.
4. Educators should assess students regularly by measuring accomplishments, modeling assessment processes, providing timely feedback, and helping students improve their self-assessment skills.
5. Faculty must accept fully the responsibility for facilitating student success.
6. To develop expertise in a discipline, a learner must develop a specific knowledge base in that field, but also acquire generic, lifelong learning skills that relate to all disciplines.
7. In a quality learning environment, facilitators of learning (teachers) focus on improving specific learning skills through timely, appropriate, and constructive interventions.
8. Mentors use specific methodologies that model the steps or activities they expect students to use in achieving their own learning goals.
9. An educational institution can continually improve its effectiveness in producing stronger learning outcomes in several ways: (1) By aligning institutional, course, and program objectives; (2) By investing in faculty development, curricular innovation, and design of performance measures; (3) By embracing an assessment culture
10. A process educator can continuously improve the concepts, processes, and tools used by doing active observation and research in the classroom.

Figure 1 Compass of Higher Education



Teaching Effectiveness Conference. It received very positive reviews and later that year it became the first official articulation of Process Education by Pacific Crest. The Ten Principles of Process Education were drafted and, with only small changes made over the years, these ten principles still exist as the core principles of Process Education. These principles are listed in Table 2.

Current Impact of Process Education

Through the application of these ten principles, Process Education is actively transforming Higher Education by empowering faculty, students, and administrators. To date, Pacific Crest has visited more than 1,800 colleges and universities, facilitated faculty development for more than 20,000 educators, and worked with more than 25,000 students in classroom situations. Pacific Crest currently offers 22 different types of professional development institutes as well as customized workshops.

To effectively meet the growing demand for these institutes and workshops, Pacific Crest has established a growing number of Regional Professional Development Centers across the United States. These Centers are dedicated to transforming the quality of teaching and learning in different areas of the country, leading to increased student retention and success at all levels. Each development center hosts three different faculty development institutes each year as part of its commitment to becoming a regional center. An individual center has the opportunity to choose its own events, in order to meet the unique needs, culture, and individual goals of each college or university. Other educational institutions in the area are invited to send participants to each institute in order to bolster the collaborative relationships among neighboring colleges (http://www.pcrest.com).

Pacific Crest's view of the interrelated processes and dynamics of Process Education has evolved over the past 25 years and is perhaps most accurately captured in the *Compass of Higher Education* (Figure 1).

Research has been conducted on each process or area, as delineated by the Compass. Much of this research is ongoing and can be seen in the *Faculty Guidebook*, a comprehensive resource on research within Process Education. The fourth edition of the *Faculty Guidebook* includes scholarship by more than 45 different authors, each of whom is dedicated to researching and sharing the most promising practices to improve teaching and learning. This edition contains 146 modules, blending theory and practice in an easy-to-use format on such topics as mentoring, assessment and evaluation, instructional design, program assessment, and creating quality learning environments. The *Faculty Guidebook*, which is also available in a web-based edition, is very accessible since it is packaged in short, comprehensive two to four page modules, thus making it easy for users to quickly absorb research, apply, and disseminate new teaching/learning knowledge and classroom innovations (http://www.pcrest.com).

Another result of research within Process Education, particularly on the critical topic of effective learning techniques, is the development of Process-Oriented Guided-Inquiry Learning (POGIL). POGIL is a technique that creates a research-based learning environment in the classroom or lab where students learn course content as well as learning process skills while working on guided-inquiry activities in small collaborative groups. The instructor facilitates this learning by asking guiding questions to teams as they work (www.pogil.org).

The individuals dedicated to the precepts of Process Education have formed a community of practice, the Academy of Process Educators. According to the Academy's web site (www.processeducation.org), the Academy "drives transformational change in education by generating, disseminating, and archiving research based on Process Education™ principles through:

- the advancement of scholarship in teaching and learning
- advocacy on key educational issues
- building an Academy research program
- the professional development of educators
- coaching and mentoring

[Furthermore], the Academy engages, supports, and collaborates with a community of educators by:

- delivering an annual conference
- producing a selective, peer-reviewed journal
- developing and endorsing position papers
- modeling key elements of Process Education
- facilitating member participation in other professional venues"

Process Education Growing into the Future

The end goal of Process Education is to create self-growers. Pacific Crest, the Academy of Process Educators, and the thousands of active users of PE are continually refining and strengthening both the development and application of Process Education. While practitioners of PE hold in common their belief in its underlying principles, the tools they use to effect the transformations that PE makes possible are as varied as the many disciplines in which they teach. It is this very diversity that makes the PE community so vibrant and such an promising arena for meaningful research and discourse.

Areas for future work in PE include developing technology that will assist educators in measuring performance, understanding what PE tools are most highly utilized by practitioners and determining if there is a pattern behind their usage, and examining and refining these tools to take into account student use and knowledge of emerging technologies.

Pacific Crest has identified that the use of technology for measuring performance as a way to help enhance performance is one of the most important transformational changes required by Higher Education. An increasing number of arenas including federal and state governments, accreditation agencies (both institutional and program) and other higher education stakeholders are requiring colleges to effectively use performance measures to document and improve student learning and growth. Pacific Crest has begun the process of creating the Performance Measurement and Enhancement System/Results Measurement System (PMES) which will collect, store, assess, and analyze measurement data to help educators and learners make better decisions for performance improvement. The data available through this system will allow PE researchers to expand and certify its inventory of measures more efficiently as well as enable the certification of new measures.

Another area for exploration and growth for process educators concerns emerging technologies. Social networking not only presents a way for friends to meet; it changes the way our youth learn. The concept of research has also begun to transition from being a primarily solitary venture to an opportunity for networks of practitioners and theorists to share resources with each other. Linear thought is being replaced by interconnected ideas. These changes trigger interesting questions: How will Process Education be transformed by this changing model? And how can PE take advantage of these changes to further facilitate learner growth and development?

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Notes

OPEN-ENDED QUESTIONS AND THE PROCESS STANDARDS

Educating students—for life, not for tests—implies incorporating open-ended questions in your teaching to develop higher-order thinking.

Wendy B. Sanchez

All societies need citizens who can solve complex problems and apply knowledge in a variety of contexts as well as citizens who can work collaboratively to solve problems and communicate solutions to mathematics education stakeholders. We must educate students to use NCTM's Process Standards (NCTM 2000) and move beyond being able to work routine exercises on standardized tests. We are not educating students for tests; we are educating them for life. All stakeholders need to see this broader picture and support teachers in this broader purpose.

As a high school mathematics teacher and mathematics teacher educator, I have used open-ended questions as part of my own teaching practice. Open-ended questions, as discussed here, are questions that can be solved or explained in a variety of ways, that focus on conceptual aspects of mathematics, and that have the potential to expose students' understanding and misconceptions. When working with teachers who are using open-ended questions with their students for the first time, I have found that they learn a considerable amount, as I did, about what their students both know and do not know—much more than what they knew before they started using open-ended questions. Teachers are almost always surprised, a little disappointed, but often excited about what they discover.

I will share some student responses from the class of a high school mathematics teacher with whom I have worked. Ms. Yoder has high expectations of her students. Her students work together to solve problems that require a high level of cognitive demand; the kind of thinking necessary to solve the problems forces students to build "connections to underlying concepts and meaning" (Stein et al. 2009, pp. 1–2). After having her students work some of the problems presented here, Ms. Yoder commented, "I was dismayed at the lack of depth and the simplicity of some students' responses. I have always felt that I teach on a conceptual level, and I do a lot of listening to students' conversations to assure myself that the level of understanding meets my hopes and expectations. . . . But I have rarely required my students to write about mathematics." After using these problems with her students, Ms. Yoder reflected, "Asking these questions made me rethink my means of assessing students."

When we think about assessment in this era of No Child Left Behind, we often think about high-stakes standardized tests, which are typically multiple-choice tests. So much of what happens in mathematics classes is focused on preparing students to succeed on these tests. As I work with teachers, they express high levels of anxiety about making sure that their students are prepared for these high-stakes tests. Mathematics education stakeholders—including teacher educators, administrators, teachers, students, and parents—need to reflect on what standardized tests can and cannot measure. Even more important, they must evaluate the educational significance of those ideas that standardized tests cannot assess.

NCTM's Process Standards—Problem Solving, Reasoning and Proof, Communication, Connections, and Representation—are difficult to assess with multiple-choice tests. For example, one aspect of the Communication Standard requires students to "communicate their mathematical thinking coherently and clearly to peers, teachers, and others" (NCTM 2000, p. 60). This standard cannot be assessed through multiple-choice questions.

If we do not teach what is not tested, what are the implications of not preparing students to meet these Process Standards? Consider the following statement by a BC Calculus student:

> My experience in the past—and not to hate on the teachers I've had—but they've never really encouraged us to think. It's all been cookie-cutter questions, even with word problems. I remember my algebra 1 teacher—she had a little trick for everything. Of course, I don't remember the trick now, and I don't remember why I was doing it. I felt like there were a lot of shortcuts, and I was never really taught why we were using them. So I memorized everything, which is what I've been doing ever since (Stockton 2010).

This student was lamenting her inability to solve a complex problem. A student capable of handling the difficult BC Calculus curriculum expressed her own disappointment that the focus of her education had been procedural.

As teachers struggle to ensure that students are able to answer questions correctly on procedural tests, many are desperate to find ways to help them remember strategies and steps to find correct solutions. However, problems that people encounter in everyday life and careers rarely require rote application of procedures.

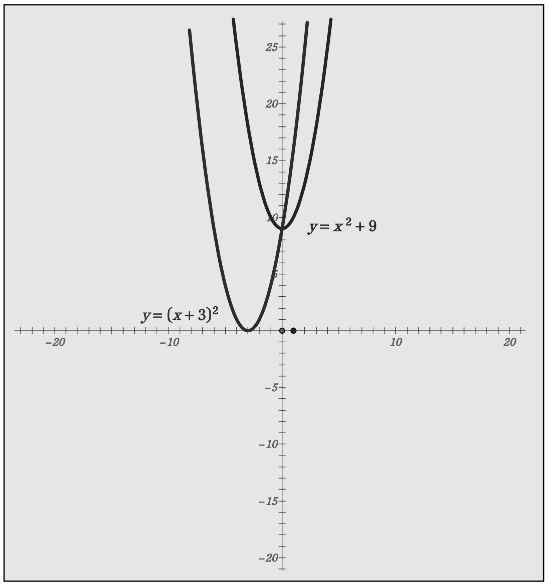

Fig. 1 Students might also argue that $y = (x + 3)^2$ and $y = x^2 + 9$ are, respectively, horizontal and vertical shifts of $y = x^2$.

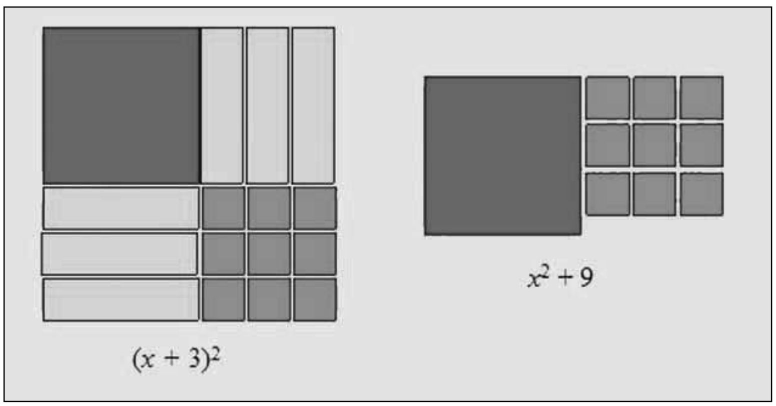

Fig. 2 Algebra tiles geometrically represent the statement $(x + 3)^2 \neq x^2 + 9$.

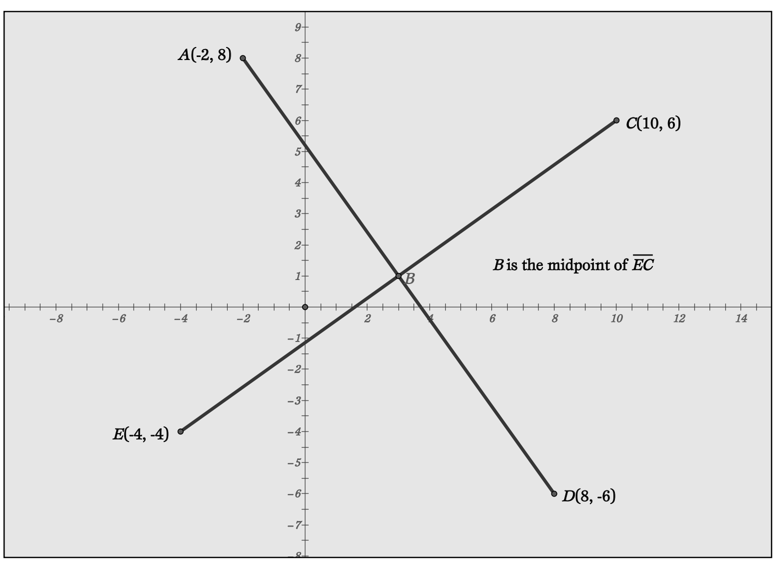

Fig. 3 Slopes, the Pythagorean theorem, congruent triangles, and dot products may all be used to show that $\angle ABC$ is a right angle.

OPEN-ENDED QUESTIONS CAN FOCUS INSTRUCTION ON PROCESS STANDARDS

Using NCTM's Process Standards as a guide, teachers can make questions more open and more focused on conceptual understanding.

Consider this traditional question:

> Expand $(x + 3)^2$.

We could revise this question in several ways. If we wanted to address the Communication Standard, we could ask students to explain how they determined their answer. We could take the question even further to incorporate other Process Standards. We could capitalize on a common student error and ask students to explain why $(x + 3)^2 \neq x^2 + 9$. Now we have expanded the question to include the Communication Standard and the Reasoning and Proof Standard. We could go even further to address the Representation Standard by asking students to give two or three different explanations of why $(x + 3)^2 \neq x^2 + 9$.

A typical first explanation that students provide is this:

> $(x + 3)^2$ means $(x + 3)(x + 3)$. I can use the distributive property to multiply these two binomials so I get $x^2 + 3x + 3x + 9$, which equals $x^2 + 6x + 9$, which is not the same as $x^2 + 9$.

Asking students for another explanation forces them to consider a different representation. For example, they might choose a numerical representation and substitute a numerical value for x. Their explanation might then be something like this:

> Let $x = 2$. $(x + 3)^2 = (2 + 3)^2 = 25$. $x^2 + 9 = 2^2 + 9 = 13$. Because $25 \neq 13$, $(x + 3)^2 \neq x^2 + 9$.

Students could also consider a graphical representation and show that the graphs of $y = (x + 3)^2$ and $y = x^2 + 9$ are different (see **fig. 1**). They could even consider the problem geometrically by using algebra tiles (see **fig. 2**).

If we teachers intentionally consider NCTM's Process Standards when writing questions, we can make sure that students are required to use the processes. With this particular question, we also counter a common student error in several ways. By seeing multiple representations, students are more likely to avoid the error later on.

What Process Standards might students use to solve the following problem?

> Use three different methods to show that $\angle ABC$ is a right angle. Explain your reasoning. (See **fig. 3** for solution.)

In solving this problem, students might use the midpoint formula to determine the coordinates of point B and then show that $AB^2 + BC^2 = AC^2$. In this way, they verify that triangle ABC is a right triangle because its sides satisfy the Pythagorean theorem and that, therefore, angle ABC is a right angle. Or, using the distance formula, students might show that $AE = AC$; then, using the side-side-side postulate, they can show that $\triangle ABC \cong \triangle ABE$. Therefore, $\angle ABE \cong \angle ABC$ because corresponding parts of congruent triangles are congruent. Because these two angles are congruent and form a linear pair, they must be right angles.

Still another way to solve this problem is to compute the slopes of $\overleftrightarrow{EC}$ and $\overleftrightarrow{AD}$ and show that their product is –1. More advanced students can demonstrate the dot product of [7, 5] (the rectangular vector from B to C) and [–5, 7] (the rectangular vector from B to A) is 0, making the two vectors orthogonal (perpendicular).

When students are required to provide multiple solutions, they often use a variety of representations. As they explain their reasoning, they are communicating. Although students need to rely on some procedural knowledge to answer this problem, they have to decide which procedures would apply to it. They are not provided with a step-by-step procedure; consequently, they are involved in problem solving as well as reasoning and proof. They are making connections among a variety of mathematical topics—slope, congruent triangles, midpoints, the distance formula, the Pythagorean theorem, and vectors.

WRITING OPEN-ENDED QUESTIONS

Open-ended questions can be written using various templates, several of which are discussed here. Teachers who are just beginning to use open-ended assessment can use these templates for creating their own questions. We provide examples of several types, and for one question of each type, we provide sample student responses.

Template 1: What's Wrong with This?

The earlier question about expanding $(x + 3)^2$ is an example of this type of question used to identify errors and misconceptions. We can ask students to identify errors and explain why they are errors. This template is useful for getting students to think critically about common misconceptions.

Some possible questions using this template follow:

1. Provide two different explanations as to why you cannot simplify the expression $(x + 3)/3$.
2. Bert was trying to graph $y = (x - 3)^2$. He said that he could simply shift the graph of $y = x^2$ three units to the left. Convince Bert that his method is incorrect.

(a)

Response A	In the graph, X≤3
Response B	sherris solution is incorrect because she misinterpreted the inequality. The correct answer is X≥3 and X≤5 because a colored dot on the segment means equal to
Response C	you know that sherri's solution is incorrect by reading the graph. The arrow coming from 3, is pointing in the lesser direction. therefore the correct solution is 5≤X≤3

(b)

Fig. 4 Sherri's solution (**a**) is incorrect. Typical student responses are shown in (**b**).

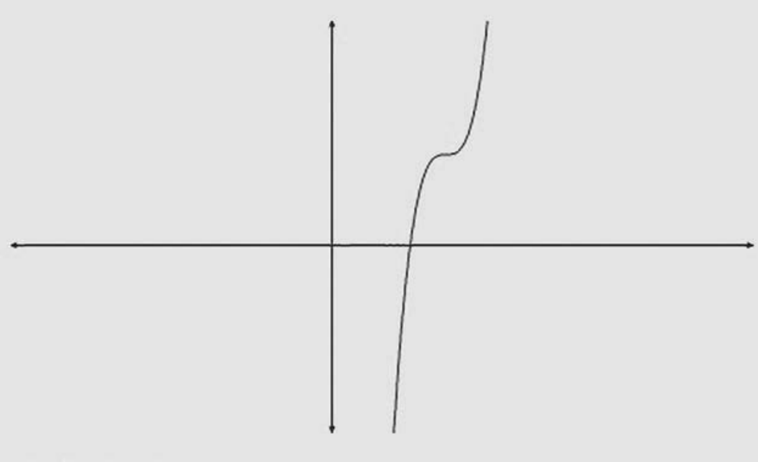

Fig. 5 Students are asked to provide a possible equation to match this graph.

3. Sherri claims that the solution set of the compound inequality $x \geq 3$ or $x \geq 5$ is shown in **figure 4**. Explain why Alaine's solution is incorrect. Provide the correct solution and explain how you know your solution is correct.

Question 3 was designed to counter the common student error of thinking that *or* always means that the arrows on the graph of a linear inequality should point in opposite directions. Of course, the correct solution set of the linear inequality is $x \geq 3$ because the *or* means one *or* the other *or* both. Therefore, any real number greater than or equal to 3 would be in the solution set.

None of the students who answered the question (even those whose solutions are not shown in **fig. 4b**) provided the correct solution. They focused on the direction of the inequality sign rather than on the meaning of the conjunction *or*. Student B appears to have some misconception about

Response A	Create a system of linear equations that has the solution (–2, 3). Explain how you determined your system. $y=(x+2)^2+3$ horizontal shift, vertical shift
Response B	$y=-2x-1 \rightarrow y=-2(-2)-1$, $y=4-1 \rightarrow y=3 \rightarrow (-2,3)$
Response C	$y=x+5 \quad y=-2x-1$; $x+5=-2x-1$; $3x=-6$; $x=-2$. I developed a system of linear equations with the solution (-2,3) then I get them equal to each other and solved for y.

Fig. 6 Students find it difficult to create a linear system when given the solution.

changing the direction of an inequality sign, an "equal to," and a "colored dot."

Template 2: Create an Example or a Situation

This form of question is similar to the form of the questions for the game show *Jeopardy*™. We give students some parameters and ask them to come up with an example or situation that fits the parameters. We give them the answer and have them come up with the question.

Some possible questions using this template follow:

1. Give a possible equation for the graph shown in **figure 5**. Explain how you determined your answer.
2. On a coordinate grid, plot and give the coordinates of four points that are the vertices of a rhombus. Explain how you know that your figure is a rhombus.
3. Create a list of ten different numbers whose median is 9. Explain how you know that the median is 9.
4. Give two complex numbers whose sum is $7 + 9i$. Explain how you know that your two numbers have the given sum.
5. Create a system of linear equations that has the solution (–2, 3). Explain how you determined your system.

The first time I used open-ended questions in my teaching, I included question 5 on an exam. Many students got every question correct except this one. The first section of the exam asked students to "solve these systems of linear equations by graphing"; the second section, to solve by substitution; the third section, to solve by elimination; and the fourth section, to solve by any method. Then I added this single open-ended question, and my students were thrown. I knew then that not only was I asking the wrong questions; I was also focusing my instruction on the wrong things. My students could follow procedures that I taught them, but they did not really know what a system of linear equations was or what a solution of a system of linear equations was.

Ms. Yoder's students' responses are informative (see **fig. 6**). Student A describes shifts of graphs of quadratic functions, whereas student B found a single line that contained the point (–2, 3). I think that students A and B would do just fine on a standardized test about systems of linear equations. Like my students who got every problem correct on my test except this one, these students might be able to answer standard questions without really understanding what a system of linear equations is. After reading these responses, however, I am much more confident that student C has a deeper understanding of systems of linear equations than either of the other two students.

Template 3: Who Is Correct and Why?

This form of open-ended question—Who is correct and why?—can be used to set up two opposing arguments. Then students can defend one or the other argument.

Some possible questions using this template follow:

1. Lucinda thinks that the grades in mathematics class should be calculated using the mean. Norm thinks that the grades should be calculated using the median. With whom do you agree and why?
2. Daniella is thinking about a particular quadratic function. Terry says that if Daniella told him the zeros of the function, he could tell her the equation of the function. Daniella maintains that Terry would need more information. Who is correct and why?
3. Candace said that if she solves the same system of linear equations as Jermaine, they could get two different answers and both be correct. Jermaine disagreed, saying that if they got two different answers, one of them must be incorrect. Who is correct and why?

Response A	Candice is correct because depending on the function if you have to take a square root there are two possible correct answers.
Response B	candice is correct because the two answers are part of the graph therefore they are correct
Response C	Jermaine, because the equations are linear, meaning only one intersect point

Fig. 7 Only one of these students fully understands the question.

Question 3 also was designed to get at the meaning of the solution of a system of linear equations. From the responses, it appears that only student C (see **fig. 7**) seems to understand the main point of the question—that two lines can intersect only in one point.

A Caution about These Templates

The templates presented here can be useful in giving teachers a place to start when writing open-ended questions, but teachers must be cautious when using them. Just because a question fits a template does not necessarily mean that the question is open ended or of high quality.

For example, we could ask the earlier question in this way:

> Jasmine solved $x + 3 = 5$ and got $x = 2$. Stuart solved $x + 3 = 5$ and got $x = 8$. Who is correct and why?

This form of the question is no different from asking the traditional question "Solve $x + 3 = 5$ for x." The formulation does not involve the conceptual underpinnings of equation solving.

PREPARATION FOR LIFE

Teachers are under more pressure than ever to ensure that students perform well on standardized tests. Consequently, many are using more multiple-choice questions to prepare their students. School districts are using benchmark testing to assess students' progress toward meeting standards and prepare them for accountability tests. These are all perfectly reasonable strategies, but mathematics education stakeholders must keep in mind the limits of these accountability tests. If we think about the purpose of schooling from a broader perspective and about preparing students to solve the kinds of problems that they will encounter in society—not just about preparing them for standardized tests—we need different strategies.

Open-ended questions can help teachers focus their instruction and assessment on NCTM's Process Standards and on reasoning and sense making, which really is the heart of mathematics. Moreover, responses to open-ended questions give teachers so much more information about students' ways of thinking and misconceptions, and these can provide important avenues for further investigation of mathematics. When students answer higher-order questions driven by the Process Standards and focused on meaning, they will be prepared for any test we give them—in school or in life.

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